4. NUMBER SYSTEMS

4.1 The Need To Quantify Very Large Quantities

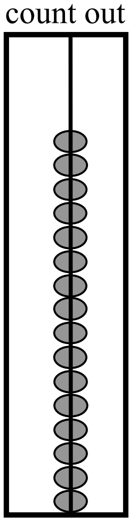
During the Paleolithic period, which ended about 10,000 BC, early man survived as a hunter and gatherer. He didn't experience much need to quantify things, and the few things he wanted to quantify—people, animals, tools, weapons, days, etc.—were usually small in quantity. However, man's need to quantify increased dramatically during the brief Neolithic period (10,000 to 5,000 BC) when he discovered how to grow food, such as grains and vegetables, and domesticate animals for food and clothing, such as sheep, goats, cows, etc. He often needed to quantify the different types of food he produced and the different types of animals he owned. Also, he invented many new tools, operated tool factories, made houses out of bricks, invented pottery, and traded his goods extensively. At first he traded goods for other goods directly, such as containers of wheat for containers of olives, but eventually money was invented as a tool of trade and exchange.

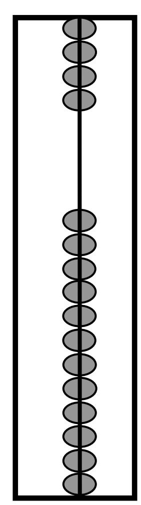
Because of this rapid acceleration of progress, people had to keep track of what they produced and traded, and how much money they earned and spent, which means they had to quantify much larger amounts of things.

Prior to this period people had names for numbers—but only small numbers, such as one, two, three, four, etc. The need for larger numbers meant that man had to invent more names for numbers and more symbols for writing the numbers. This was a major challenge. If he were to invent unique names and symbols unrelated to each other, such as one, two, three, etc., or 1, 2, 3, etc., he would have to invent and memorize a lot of unique, unrelated names and symbols, which is tedious. Eventually, man realized that he needed to simplify the naming of numbers and the symbols used to write numbers in order to make them easy to remember, record, and quantify with. How?

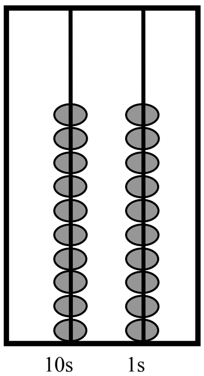
4.2 The Abacus: A Revolutionary Invention

When people started to produce large quantities of food and other goods, and trade them in the marketplace, a quick and reliable method of quantifying large quantities was desperately needed. A seller didn't want customers to wait in line too long; otherwise they might leave and go to his competitors. Also, a device was needed whereby after quantifying one group of things he could quickly start from zero again and perform a new quantification. For example, a seller might have to count out 100 figs for one customer and then count 200 figs for the next customer. Around 3000 BC, after the birth of civilization in Mesopotamia, some clever person invented a revolutionary counting device. Imagine placing a large number of beads on a string tied to a frame as shown on the left, only with many more beads than shown. Imagine it lying flat on a table so that gravity doesn't cause the beads to move back down. To start off at zero all the beads are pushed to the bottom. Assuming that each bead has a value of "one," one can count by sliding beads up the string, one at a time, as shown on the right to represent a counting of four. When starting a new count one just moves all the beads back down. The problem with this device is that you need a lot of beads to count large quantities. For example, you would need 100 beads to count up to 100, and 1000 beads to count up to 1000. Can you think of a way to count larger numbers without having to use many beads? Hint: Why assign each bead the same value?

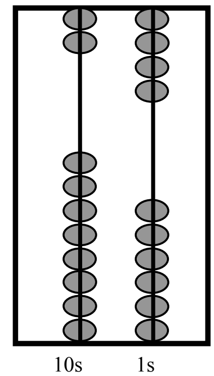




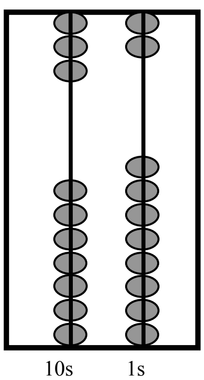
Why not have some of the beads equal to, say, 10 units each? But how would you keep track of the value of each bead? Some clever person around 3000 BC found the solution. Instead of having one column of 100 beads, why not make two columns of ten beads each where the beads in the first column have a value of one and the beads in the second column have a value of 10?

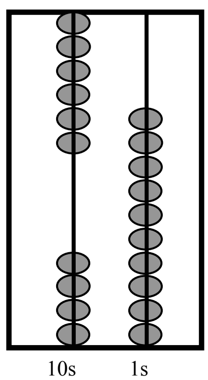


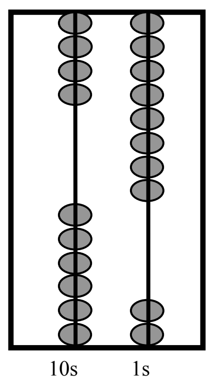
The left diagram shows the zero position. To start counting you move the beads in the singles (rightmost) column up until you use up the 10 beads. At this point you move the 10 beads down again and move up one bead in the 10s column since 10 singles equals 1 ten. Now you can continue counting with singles until you use up the ten beads again. At this point, you move them down again and move up one more ten. You can continue this procedure up to 100 (actually 110, but we'll be satisfied with 100 for now). The device on the right shows a quantity of 2 tens and 4 singles which equals 24.

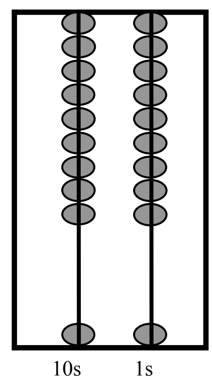


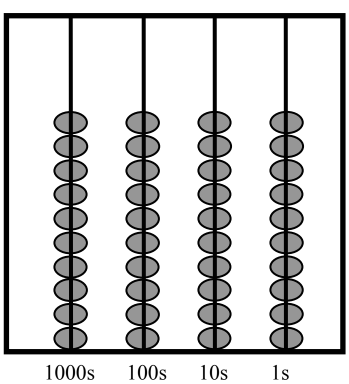
a) Below each device below, write the quantity shown by the device using Hindu-Arabic numerals.











Hence, by creating an extra column of beads, and assigning them a value of ten each, called its **place value**, we can now count to 100 with only 20 beads, instead of 100 beads, thereby saving 80 beads. Can you infer what we must do to create a device that counts to 1000? We can follow the same pattern by adding another column of ten beads and assigning each bead a place value of 100. Likewise, as shown on your left, we can we can add a fourth column of ten beads and assign each bead a place value of 1000 to create a device that counts to 10000 (actually 11,110). Hence, instead of 10,000 beads to count to 10,000 we only need 40 beads, which is a significant savings on beads, which were harder to make long ago.

This ingenious and beneficial device is called an **abacus**. You have probably seen one before, but rotated a quarter turn so that the beads form (horizontal) rows, not columns. That way, if you hold the abacus upright, then the beads will stay in the position you moved them, unaffected by gravity. However, for reasons you'll soon discover, it's more beneficial for learning math to show the strings of beads oriented vertically, not horizontally. Imagine it sitting on a table and you are looking down at it. That way, gravity won't be a factor.

Abacus is a Latin word that originates from the Greek word **abax** meaning "table," perhaps because it sits on a table and looks like a table of information. There are different types of **abaci**, and

When dividing large numbers by large numbers, one uses the procedure for long division presented earlier. When one gets to the last step and finds that the number doesn't divide evenly, one just takes what's left over as the remainder. Consider the following division: 11597÷105. The first three steps are the same as in the previous section. At the end of the third step (3), we used up the last digit in the dividend, 7, to get 47. However, 47 is less than our divisor, 105. Hence, 47 is our remainder and the division is basically complete. The remaining step is to place the 47 as the remainder to get R47. So: 11597÷105=110R47.

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(1)              (2)              (3)              (4)

    1               11              110              110R47
105)11597       105)11597       105)11597       105)11597
    105             105             105             105
     10              109             109             109
                     105             105             105
                       4              47              47
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b) Perform the two divisions below in the space provided.

113442÷213

192275÷487

c) Check your answer to the above two divisions by using multiplication in the spaces below.

Before proceeding, answer all the questions for Sections 9.7 in the exercise section.

4.1 The Need To Quantify Very Large Quantities

1) During the Neolithic period of history, what two major discoveries took place, both involving food production, that greatly increased man's need to quantify things?

2) Why did these discoveries increase man's need to quantify things?

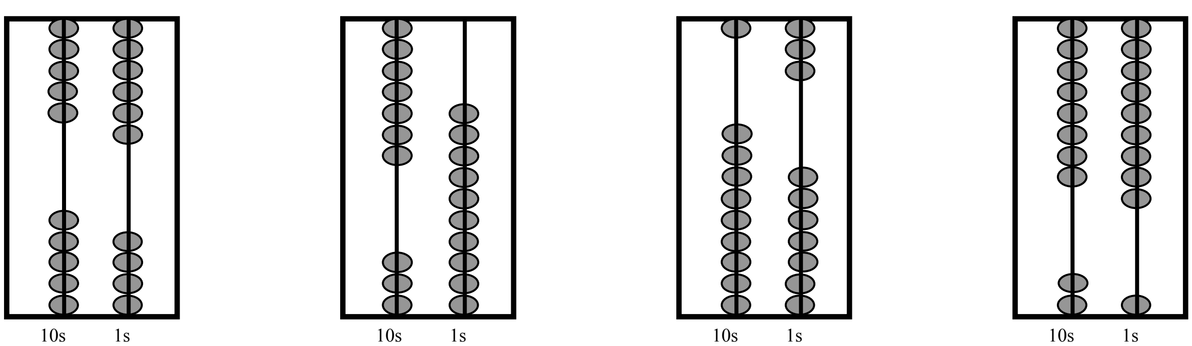
3) What challenge arose when dealing with larger numbers?

4.2 The Abacus: A Revolutionary Invention

1) When using beads on a string to count, what problem arises when only one column exists?

2) Instead of using a single-column device where 100 beads are needed to count to 100, what configuration of beads was invented to allow the same counting with only 20 beads?

3) Below each abacus, use Hindu-Arabic numerals to write the quantity shown.



4) Complete the following division table.

21 √‾17246	34 √‾23980	40 √‾19623	45 √‾41656
51 √‾26682	56 √‾47188	62 √‾50262	68 √‾49364
28 √‾25105	37 √‾25195	48 √‾25326	73 √‾52040
78 √‾66847	80 √‾68840	93 √‾64906	98 √‾95717

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